Engineering Mathematics
Diploma 1st year

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Circle

Definition:

A circle is the locus of a point which moves on a plane in such a way that its distance from a fixed point is always constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

In the Fig 3.23, $P(x, y)$ is the moving point, C is the centre and CP is the radius.

1. **Standard form** (Equation of a Circle with given centre and radius)

Let C(α, β) be the centre of the circle and radius of the circle be 'r'

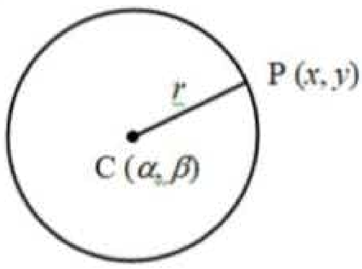
Let $P(x, y)$ be any point on the circumference of the circle.

Then,

$$CP = r$$

By distance formula,

$$\sqrt{(x-\alpha)^2 + (y-\beta)^2} = r$$



Or, $(x-\alpha)^2 + (y-\beta)^2 = r^2$

Which is the equation of the circle having centre at (α, β) and radius 'r', which is known as standard form of equation of a circle.

Note: If the centre of the circle is at origin, $(0, 0)$ and radius is 'r', then the above standard equation of the circle reduces to $x^2 + y^2 = r^2$.

Some Particular Cases:

The standard equation of the circle with centre at $C(\alpha, \beta)$ and radius r, is

$$(x-\alpha)^2 + (y-\beta)^2 = r^2 \qquad (1)$$

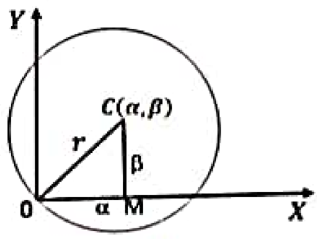


Fig. 3.24

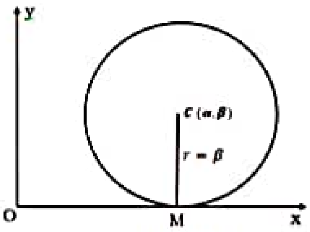


Fig 3.25

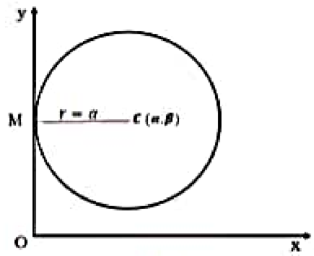


Fig 3.26

(i) **When the circle passes through the origin**

From the Fig 3.24, In right angle triangle ΔOCM,

$OC^2 = OM^2 + CM^2$ i.e. $r^2 = \alpha^2 + \beta^2$

Then eqn (1) becomes,

$(x-\alpha)^2 + (y-\beta)^2 = \alpha^2 + \beta^2$

Or, $x^2 + y^2 - 2\alpha x - 2\beta y = 0$

(ii) **When the circle touches x – axis**

In the Fig 3.25, Here, $r = \beta$

Hence, the eqn (1) of the circle becomes,

$(x-\alpha)^2 + (y-\beta)^2 = \beta^2$

Or, $x^2 + y^2 - 2\alpha x - 2\beta y + \alpha^2 = 0$

(iii) **When the circle touches y – axis**

In the Fig 3.26, Here, $r = \alpha$

Hence, the eqn (1) of the circle becomes,

$(x-\alpha)^2 + (y-\beta)^2 = \alpha^2$

Or, $x^2 + y^2 - 2\alpha x - 2\beta y + \beta^2 = 0$

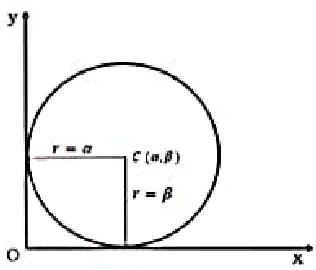


Fig 3.27

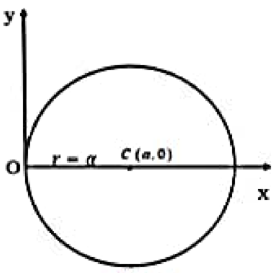


Fig 3.28

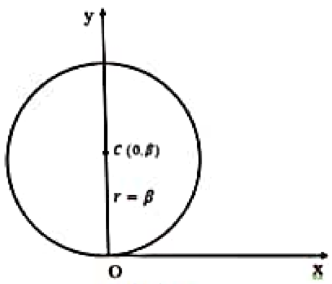


Fig 3.29

(iv) When the circle touches both the axes

In the Fig, 3.27, Here, $\alpha = \beta = r$

Hence, the eqn (1) of the circle becomes,

$$(x - r)^2 + (y - r)^2 = r^2$$

Or, $x^2 + y^2 - 2rx - 2ry + r^2 = 0$

(v) When the circle passes through the origin and centre lies on x- axis

Here, $\alpha = r$ and $\beta = 0$

Hence, the eqn (1) of the circle becomes,

$$(x - r)^2 + (y - 0)^2 = r^2$$

Or, $x^2 + y^2 - 2rx = 0$

(vi) When the circle passes through the origin and centre lies on y- axis

Here, $\alpha = 0$ and $\beta = r$

Hence, the eqn (1) of the circle becomes,

$$(x - 0)^2 + (y - r)^2 = r^2$$

Or, $x^2 + y^2 - 2ry = 0$

Some Solved Problems

Q- 1: Find equation of the circle which has centre at (2, 3) and radius is 4 .

Sol:

According to the standard form, the equation of circle with centre at (α, β) and radius r is

$$(x - \alpha) + (y - \beta)^2 = r^2$$

$\therefore$ Equation of the circle with centre at (2, 3) and radius 4 is,

$$(x - 2)^2 + (y - 3)^2 = (4)^2$$

Or, $x^2 + y^2 - 4x - 6y + 13 = 16$

Or, $x^2 + y^2 - 4x - 6y - 3 = 0$

Q- 2: Find equation the circle which has centre at (1, 4) and passes through a point (2, 6).

Sol:

Given $C(1, 4)$ be the centre and r be the radius of the circle. The circle passes through the point $P(2, 6)$

$\therefore$ $PC = r$

Or, $\sqrt{(2-1)^2 + (6-4)^2} = r$ (By using distance formula)

Or, $\sqrt{1+4} = r$

Or, $r = \sqrt{5}$

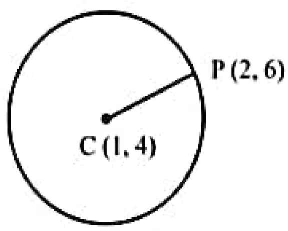


Fig 3.30

By using standard form of the circle,

Equation of the circle with centre at C(1,4) and radius $\sqrt{5}$ is

$$(x - 1)^2 + (y - 4)^2 = \left(\sqrt{5}\right)^2$$

Or, $x^2 + y^2 - 2x - 8y + 1 + 16 = 5$

Or, $x^2 + y^2 - 2x - 8y + 12 = 0$

Q- 3: Find equation of the circle whose centre is at (5, 5) and touches both the axis.

Sol:

The centre of the given circle is at (5, 5).

Since the circle touches both the axes,

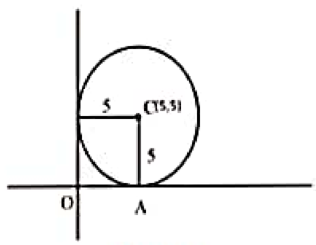


Fig 3.31

$\therefore$ radius, $r = 5$

According to the standard form,

$\therefore$ Equation of the circle with centre at C(5,5) and radius $r = 5$ is

$$(x-5)^2 + (y-5)^2 = (5)^2$$

Or, $x^2 + y^2 - 10x - 10y + 25 = 0$

Q- 4: If the equation of two diameters of a circle are $x - y = 5$ and $2x + y = 4$, and the radius of the circle is 5, find the equation of the circle.

Sol:

Let the diameters of the circle be AB and LM, whose equations are respectively,

$x - y = 5$ (1)

and $2x + y = 4$ (2)

Since, the point of intersection of any two diameters of a circle is its centre and by solving the equations of two diameters we find the co-ordinates of the centre.

$\therefore$ Solving eqns. (1) and (2), we get $x = 3$ and $y = -2$

Therefore, co-ordinates of the centre are $(3, -2)$ and radius is 5.

Hence, equation of required circle is

$$(x-3)^2 + (y+2)^2 = 5^2$$

Or, $x^2 + y^2 - 6x + 4y + 9 + 4 = 25$

Or, $x^2 + y^2 - 6x + 4y - 12 = 0$

Q-5: Find the equation of a circle whose centre lies on positive direction of $y-$ axis at a distance 6 from the origin and whose radius is 4.

Sol:

Given, the centre of the circle lies on positive y-axis at a distance 6 units from origin.

$\therefore$ The centre of the circle lies at the point $C(0, 6)$.

Hence, equation of the circle with centre at C(0, 6) and radius '4' is

$$(x-0)^2 + (y-6)^2 = 4^2$$

Or, $x^2 + y^2 - 12y + 36 = 16$

Or, $x^2 + y^2 - 12y + 20 = 0$

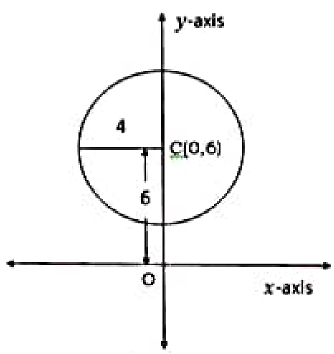


Fig 3.32

2. General form

Theorem: The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ always represents a circle whose centre is at $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$.

Proof:

The given equation is $x^2 + y^2 + 2gx + 2fy + c = 0$

Or, $(x^2 + 2gx + g^2) + (y^2 + 2fy + f^2) = g^2 + f^2 - c$

Or, $(x+g)^2 + (y+f)^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2$

Or, $\{x - (-g)\}^2 + \{y - (-f)\}^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2$

Which is in the standard form (i.e. $(x-\alpha)^2 + (y-\beta)^2 = r^2$) of the circle with centre at (α, β) and radius 'r'.

Hence the given equation $x^2+y^2+2gx+2fy+c=0$ represents a circle whose centre is $(-g,-f)$ i.e. $\left(-\frac{1}{2}\text{ coeff. of } x,\ \ -\frac{1}{2}\text{ coeff. of } y\right)$.

And radius $=\sqrt{g^2+f^2-c}=\sqrt{\left(-\frac{1}{2}\text{ coeff. of } x\right)^2+\left(-\frac{1}{2}\text{ coeff. of } y\right)^2-\text{constant term}}$

Notes : Characteristics of the general form of equation of circle

The characteristics of general form $x^2+y^2+2gx+2fy+c=0$ of a circle are

i. It is quadratic(of second degree) both in x and y.
ii. Coefficient of x^2 = Coefficient of y^2.
iii. It is independent of the term xy, i.e. there is no term containing xy.
iv. Contains three arbitrary constants i.e. g, f and c.

Note : To find the centre and radius of the circle, which is in the form $ax^2+ay^2+2gx+2fy+c=0$, where $a\neq 0$,

Divide both sides of the equation by coefficient of x^2 or y^2 (i.e. a) to get

$x^2+y^2+\frac{2g}{a}x+\frac{2f}{a}y+\frac{c}{a}=0$,

Which is in the general form of the circle

Hence, the co-ordinates of the centre are $\left(-\frac{1}{2}\text{ coeff. of } x,\ \ -\frac{1}{2}\text{ coeff. of } y\right)$

$$=\left(-\frac{1}{2}\frac{2g}{a},\ \ -\frac{1}{2}\frac{2f}{a}\right)=\left(-\frac{g}{a},\ \ -\frac{f}{a}\right)$$

and radius= $\sqrt{\left(-\frac{1}{2}\text{ coeff. of } x\right)^2+\left(-\frac{1}{2}\text{ coeff. of } y\right)^2-\text{constant term}}$

$$=\sqrt{\frac{g^2}{a^2}+\frac{f^2}{a^2}-\frac{c}{a}}$$

Example-1:

Let the equation of a circle be $25x^2+25y^2-30x-10y-6=0$

To find the centre and radius of the above circle, divide by coefficient of x^2 i.e. 25, as

$$x^2+y^2-\frac{30}{25}x-\frac{10}{25}y-\frac{6}{25}=0$$

Or, $x^2+y^2-\frac{6}{5}x-\frac{2}{5}y-\frac{6}{25}=0$

Or, $x^2+y^2-\frac{6}{5}x-\frac{2}{5}y-\frac{6}{25}=0$

Or, $x^2+y^2+2\left(-\frac{3}{5}\right)x+2\left(-\frac{1}{5}\right)y+\left(-\frac{6}{25}\right)=0$,

which is the general form of circle with centre at $(-g,-f)=\left(\frac{3}{5},\frac{1}{5}\right)$ and radius

$$=\sqrt{\left(-\frac{3}{5}\right)^2+\left(-\frac{1}{5}\right)^2-\left(-\frac{6}{25}\right)}=\frac{4}{5}$$

Example-2:

Consider the equation of a circle $x(x+y-6)=y(x-y+8)$

Or, $x^2+xy-6x=xy-y^2+8y$

Or, $x^2+y^2-6x-8y=0$,

Which, is in the general form of circle.

$\therefore$ Centre $=(-g,-f)=\left(-\frac{1}{2}\text{ coeff. of } x,\ \ -\frac{1}{2}\text{ coeff. of } y\right)=\left(-\frac{1}{2}(-6),-\frac{1}{2}(-8)\right)=(3,4)$

and radius = $\sqrt{g^2+f^2-c} = \sqrt{\left(-\frac{1}{2}\text{ coeff. of } x\right)^2 + \left(-\frac{1}{2}\text{ coeff. of } y\right)^2 - \text{constant term}}$

$$= \sqrt{\left(-\frac{1}{2}(-6)\right)^2 + \left(-\frac{1}{2}(-8)\right)^2 - 0} = \sqrt{9+16-0} = 5$$

Example-3:

Let the equation of the circle be $x^2 + y^2 + 4x + 6y + 2 = 0$.

Compare this given equation with the general equation of the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$.

Here, $2gx = 4x$, $2fy = 6y$, and $c = 2$

So, $g = 2$, $f = 3$ and $c = 2$

Now, Centre is at (-g, -f) = (-2, -3) and $r = \sqrt{g^2+f^2-C} = \sqrt{4+9-2} = \sqrt{11}$

Some Solved Problems:

Q-1 : Determine which of the circles $x^2 + y^2 - 3x + 4y = 0$ and $x^2 + y^2 - 6x + 8y = 0$ is greater.

Sol:

The equations of two given circles are

$C_1: x^2 + y^2 - 3x + 4y = 0$

and $C_2: x^2 + y^2 - 6x + 8y = 0$

In 1st circle C_1, $g = \frac{3}{2}$, $f = 2$, $c = 0$

radius = $r_1 = \sqrt{g^2+f^2-c} = \sqrt{\left(\frac{3}{2}\right)^2 + 2^2 - 0} = \sqrt{\frac{9}{4}+4} = \frac{5}{2}$

Similarly, In 2nd circle C_2, $g = 3$, $f = 4$, $c = 0$

radius = $r_2 = \sqrt{g^2+f^2-c} = \sqrt{3^2+4^2-0} = \sqrt{9+16} = 5$

Since, $r_1 < r_2$, So, the 2nd circle $C_2: x^2 + y^2 - 6x + 8y = 0$ is greater.

Q-2: Find the equation of the circle concentric with the circle $x^2 + y^2 - 4x + 6y + 10 = 0$ and having radius 10 units.

Sol:

The coordinates of the centre of the given circle, $x^2 + y^2 - 4x + 6y + 10 = 0$,are $\left(-\frac{1}{2}\text{ coeff. of } x,\ -\frac{1}{2}\text{ coeff. of } y\right) = (2, -3)$.

Since the required circle is concentric with the above circle , the centre of the required circle and above given circle are same.

∴ Centre of the required circle is at $(2, -3)$.

Hence, the equation of the required circle with centre at $(2, -3)$ and radius 10 is

$$(x-2)^2 + (y+3)^2 = (10)^2$$

Or, $x^2 + y^2 - 4x + 6y - 87 = 0$

Q-3: Find the equation of the circle whose centre is at the point (4, 5) and passes through the centre of the circle: $x^2 + y^2 - 6x + 4y - 12 = 0$.

Sol:

The co-ordinates of the centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ are

$$C_1\left(-\frac{1}{2}\text{ coeff. of } x,\ -\frac{1}{2}\text{ coeff. of } y\right) = C_1(3, -2).$$

Therefore, the required circle passes through the point $C_1(3, -2)$.

Given, the centre of the required circle is at $C(4, 5)$

$\therefore$ radius of the required circle= $CC_1 = \sqrt{(4-3)^2 + (5+2)^2} = \sqrt{1+49} = \sqrt{50}$

Hence, the equation of the required circle with centre at $C(4,5)$ and radius '$\sqrt{50}$' is

$$(x-4)^2 + (y-5)^2 = (\sqrt{50})^2$$

Or, $x^2 + y^2 - 8x - 10y - 9 = 0$

Q-4: Find the equation of the circle concentric with the circle $4x^2 + 4y^2 - 24x + 16y - 9 = 0$ and having its area equal to 9π sq. units.

Sol:

The equation of given circle is $4x^2 + 4y^2 - 24x + 16y - 9 = 0$

Or, $x^2 + y^2 - 6x + 4y - \frac{9}{4} = 0$

$\therefore$ Centre $\left(-\frac{1}{2}\text{ coeff. of } x, \ -\frac{1}{2}\text{ coeff. of } y\right) = (3,-2)$.

Since the required circle is concentric with the above circle , the centre of the required circle and above given circle are same.

$\therefore$ Centre of the required circle is $(3,-2)$ and let its radius be 'r'

Again, Given Area of the required circle $= 9\pi$

Or, $\pi r^2 = 9\pi$

Or, $r = 3$ units

Therefore, the equation of the required circle with centre at $(3,-2)$ and radius '3' is

$$(x-3)^2 + (y+2)^2 = (3)^2$$

Or, $x^2 + y^2 - 6x + 4y + 4 = 0$

Q-5: Find the equation of the circle concentric with the circle $2x^2 + 2y^2 + 8x + 12y - 25 = 0$ and having its circumference equal to 6π sq. units.

Sol:

The equation of given circle be $2x^2 + 2y^2 + 8x + 12y - 25 = 0$

Or, $x^2 + y^2 + 4x + 6y - \frac{25}{2} = 0$

$\therefore$ centre $= \left(-\frac{1}{2}\text{ coeff. of } x, \ -\frac{1}{2}\text{ coeff. of } y\right) = (-2,-3)$.

Since the required circle is concentric with the above circle, the centre of the required circle and above given circle are same.

$\therefore$ Centre of the required circle is $(-2,-3)$ and let its radius be 'r'

Again, Given, circumference of the required circle $= 6\pi$

Or, $2\pi r = 6\pi$

Or, $r = 3$ units

Therefore, the equation of the required circle with centre at $(-2,-3)$ and radius '3' is

$$(x+2)^2 + (y+3)^2 = 3^2$$

Or, $x^2 + y^2 + 4x + 6y + 4 = 0$

Equation of a Circle satisfying certain given conditions

The general equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ involves three unknown quantities g, f and c called the arbitrary constants. These three constants can be determined from three equations involving g, f and c. These three equations can be obtained from three independent given conditions. We find the values of g, f and c by solving these three equations, and putting these values in the equation of circle, we get the required equation of circle.

Some Solved Problems:

Q-1: Find equation of the circle passes through the points (0, 0), (1, 0) and (0, 1).

Sol:

Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad (1)$$

Since, the circle (1) passes through the points (0, 0), (1, 0) and (0, 1). We have,

$$0 + 0 + 0 + 0 + c = 0, \quad \text{Or,} \quad c = 0 \quad (2)$$

$$1 + 0 + 2g + 0 + 0 = 0, \quad \text{Or,} \quad g = -1/2 \quad (3)$$

and, $0 + 1 + 0 + 2f + 0 = 0, \quad \text{Or,} \quad f = -1/2 \quad (4)$

Putting the values of g, f and c in equation (1), we get

$$x^2 + y^2 + 2\left(\frac{-1}{2}\right)x + 2\left(\frac{-1}{2}\right)y + 0 = 0$$

Or, $x^2 + y^2 - x - y = 0$, is the equation of required circle.

Q-2: Find the equation of the circle passes through the points $(0, 2)$, $(3, 0)$ and $(3, 2)$. Also, Find the centre and radius.

Sol:

Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad (1)$$

Since, the circle (1) passes through the points (0, 2), (3, 0) and (3, 2) i.e these points lie on the circle (1), we have,

$\therefore \quad 0 + 4 + 0 + 4f + c = 0$,

Or, $4f + c = -4 \quad (2)$

$9 + 0 + 6g + 0 + c = 0$

Or, $6g + c = -9 \quad (3)$

and, $9 + 4 + +6g + 4f + c = 0$

Or, $6g + 4f + c = -13 \quad (4)$

On solving equations (2), (3) and (4),

Eqns (2)+(3) : $\quad 6g + 4f + 2c = -13 \quad (5)$

Eqns (5)-(4) : $\quad c = 0$

Putting the value of c in (2) and (3), we get

$4f = -4 \quad$ Or, $f = -1$

$6g = -9 \quad$ Or, $g = -\frac{3}{2}$

Putting the values of g, f and c in the general eqn of circle (1), we get

$$x^2 + y^2 + 2\left(-\frac{3}{2}\right)x + 2(-1)y + 0 = 0$$

Or, $x^2 + y^2 - 3x - 2y = 0$, is the equation of required circle.

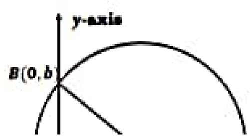
Now, the centre of the circle= $(-g, -f) = \left(\frac{3}{2}, 1\right)$

and radius= $\sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{4} + 1 - 0} = \frac{\sqrt{13}}{2}$

Q-3: Find the equation of the circle which passes through the origin and cuts off intercepts a and b from the positive parts of the axes.

Sol:

Let the equation of the circle be



$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Since, the circle passes through the origin
and cuts off the intercepts a and b from the positive axes.
So, the circle passes through the points
$O(0,0)$, $A(a,0)$ and $B(0,b)$. We have,

$0 + 0 + 0 + 0 + c = 0,$ Or, $c = 0$ (2)

$a^2 + 0 + 2ag + 0 + 0 = 0,$ Or, $g = -a/2$ (3)

and $0 + b^2 + 0 + 2bf = 0 = 0,$ Or, $f = -b/2$ (4)

Putting the values of $g,\ f$ and c in the equation of circle (1), we get

$x^2 + y^2 - ax - by = 0$ is the equation of required circle.

Q-4: Prove that the points (2, –4), (3, –1), (3, –3) and (0, 0) are concyclic.

Sol:

Note : To prove that four given points are concyclic (i.e. four points lie on the circle), we find the equation of the circle passing through any three given points and show that the fourth point lies on it.

Let the equation of the circle passing through the points (0, 0), (2, –4) and (3, –1) be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Since the point (0, 0) lies on circle (1),we have,

$0 + 0 + 0 + 0 + c = 0,$ Or, $c = 0$ (2)

Again, since the point (2, –4) lies on circle (1),we have,

$4 + 16 + 4g - 8f + 0 = 0,$

Or, $4g - 8f = -20,$

Or, $g - 2f = -5$ (3)

Also, since the point (3, –1) lies on circle (1),we have,

$9 + 1 + 6g - 2f + 0 = 0,$

Or, $6g - 2f = -10,$

Or, $3g - f = -5$ (4)

Now, solving equations (3) and (4), we get

$g = -1$ and $f = 2$

Putting the values of $g,\ f$ and c in the equation of circle (1), we get

$x^2 + y^2 - 2x + 4y = 0,$ (5)

is the equation of circle. Now, to check the 4th point (3, –3) lies on the circle (5),we put $x = 3$ and $y = -3$ in eqn (5),

$9 + 9 - 6 - 12 = 0,$

Therefore, the point (3, –3) satisfies the equation of circle (5) and lies on the circle.
Hence, the given points are concyclic.

Q-5: Find the equation of circle which passes through (3, –2), (–2, 0) and has its centre on the line $2x - y = 3$.

Sol:

Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad (1)$$

Since, the circle (1) passes through the points (3, –2) and (–2, 0) i.e these points lie on the circle (1),we have

$9 + 4 + 6g - 4f + c = 0$

Or, $6g-4f+c=-13$ (2)

and $4+0+4g+0+c=0$

Or, $4g+c=-4$ (3)

Again, the centre $(-g,-f)$ of circle (1) lies on the line $2x-y=3$

$\therefore$ $-2g+f=3$ (4)

Now, solving equations (2), (3) and (4), we get

eqn(2)-eqn(3) : $2g-4f=-9$ (5)

eqn(4)+eqn(5): $-3f=-6,$ Or, $f=2$

i.e $-2g+2=3$ Or, $g=-1/2$

i.e $4(-1/2)+c=-4,$ Or, $c=-2$

Putting the values of $g,$ f and c in the equation of circle (1), we get

$$x^2+y^2+2\left(-\frac{1}{2}\right)x+2\,(2)y+(-2)=0$$

Or, $x^2+y^2-x+4y-2=0$ is the equation of required circle.

Q-6: Find the equation of the circle circumscribing the triangle ΔABC whose vertices are A(1, –5), B(5, 7) and C(–5, 1).

Sol:

Let the equation of the circle be

$$x^2+y^2+2gx+2fy+c=0 \quad (1)$$

Since, the circle circumscribing the triangle ΔABC with vertices A(1, –5), B(5, 7) and C(–5, 1), So, the circle (1) passes through the points A(1, –5), B(5, 7) and C(–5, 1).

Therefore,

$1+25+2g-10f+c=0,$

Or, $2g-10f+c=-26$ (2)

$25+49+10g+14f+c=0$

Or, $10g+14f+c=-74$ (3)

and $25+1-10g+2f+c=0$

Or, $-10g+2f+c=-26$ (4)

On solving equations (2), (3) and (4), we get

eqn(3)-eqn(2): $8g+24f=-48,$ Or, $g+3f=-6$ (5)

eqn(3)-eqn(4): $20g+12f=-48,$ Or, $5g+3f=-12$ (6)

eqn(5)-eqn(6): $-4g=6,$ Or, $g=-\frac{3}{2}$

i.e $-\frac{3}{2}+3f=-6,$ Or, $f=-\frac{3}{2}$

i.e $2\left(-\frac{3}{2}\right)-10\left(-\frac{3}{2}\right)+c=-26$, Or, $c=-38$

Putting the values of $g,$ f and c in the equation of circle (1),

$$x^2+y^2+2\left(-\frac{3}{2}\right)x+2\left(-\frac{3}{2}\right)y+(-38)=0$$

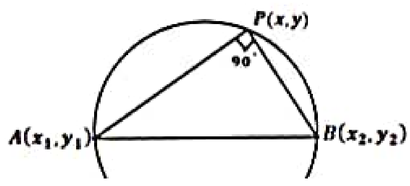
Or, $x^2+y^2-3x-3y-38=0$ is the equation of required circle.

3. Diameter form (Equation of a circle with given end points of a diameter)

Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be two end points of diameter AB of a circle. Let $P(x,y)$ be any point on the circle.

Join AP and BP. $\therefore \angle APB=90^\circ$

($\because$ An angle on a semi-circle is right angle)



Now, slope of $AP=\frac{y-y_1}{x-x_1}$ and slope of $BP=\frac{y-y_2}{x-x_2}$

Since $AP \perp BP$

By condition of perpendicularity, the product of their slopes $= -1$

Or, $\frac{y-y_1}{x-x_1} \frac{y-y_2}{x-x_2} = -1$

Or. $(y-y_1)(y-y_2) = -(x-x_1)(x-x_2)$

Or. $(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$

is the equation of circle with end points of a diameter (x_1, y_1) and (x_2, y_2), which is known as the diameter form of equation of circle.

Some Solved Problems:

Q- 1 : Find equation of a circle whose end points of a diameter are (1, 2) and (–3, –4).

Sol:

We know that, equation of the circle with end points (x_1, y_1) and (x_2, y_2) of a diameter is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$$

Given, the end points of diameter are (1, 2) and (–3, –4).

Therefore, the equation of the circle is

$$(x-1)(x+3)+(y-2)(y+4) = 0$$

Or, $x^2 + 2x - 3 + y^2 + 2y - 8 = 0$

Or, $x^2 + y^2 + 2x + 2y - 11 = 0$

Q-2: Find the equation of the circle passing through the origin and making intercepts 4 and 5 on the axes of co-ordinates.

Sol:

Let the intercepts be $OA = 4$ and $OB = 5$.

∴ The co-ordinates of A and B are $(4,0)$ and $(0,5)$ respectively.

Since $\angle AOB = \frac{\pi}{2}$, therefore AB is the diameter.

According to the diameter form,

Equation of the circle with end points $(4,0)$ and $(0,5)$ of diameter AB is

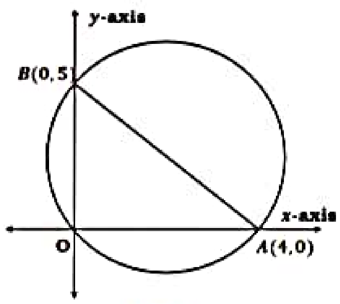


Fig 3.35

$$(x-4)(x-0)+(y-0)(y-5) = 0$$

Or, $x^2 + y^2 - 4x - 5y = 0$